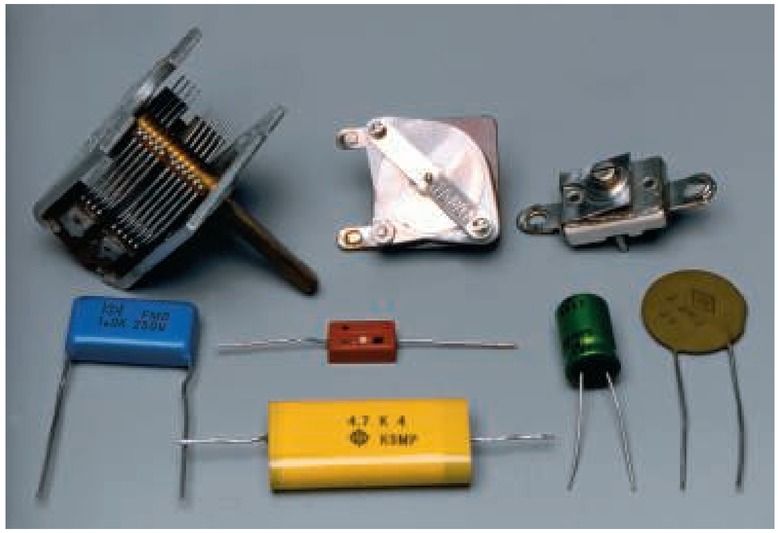
Capacitors come in a wide range of sizes and shapes, only a few of which are shown here. A capacitor is basically two conductors that do not touch, and which therefore can store charge of opposite sign on its two conductors. Capacitors are used in a wide variety of circuits, as we shall see in this Chapter.



Tom Pantages

Capacitance, Dielectrics, Electric Energy Storage

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!

[*Don't worry about getting the right answer now—the idea is to get your preconceived notions out on the table.*]

A fixed potential difference V exists between a pair of close parallel plates carrying opposite charges $+Q$ and $-Q$. Which of the following would not increase the magnitude of charge that you could put on the plates?

(a) Increase the size of the plates.
(b) Move the plates farther apart.
(c) Fill the space between the plates with paper.
(d) Increase the fixed potential difference V.
(e) None of the above.

This Chapter deals first of all with an important device, the capacitor, which is used in many electronic circuits. We will also discuss electric energy storage and the effects of an insulator, or dielectric, on electric fields and potential differences.

1 Capacitors

PHYSICS APPLIED
Uses of capacitors

A **capacitor** is a device that can store electric charge, and normally consists of two conducting objects (usually plates or sheets) placed near each other but not touching. Capacitors are widely used in electronic circuits. They store charge for later use, such as in a camera flash, and as energy backup in computers if the power fails. Capacitors also block surges of charge and energy to protect circuits.

Note: Sections marked with an asterisk (*) may be considered optional by the instructor.

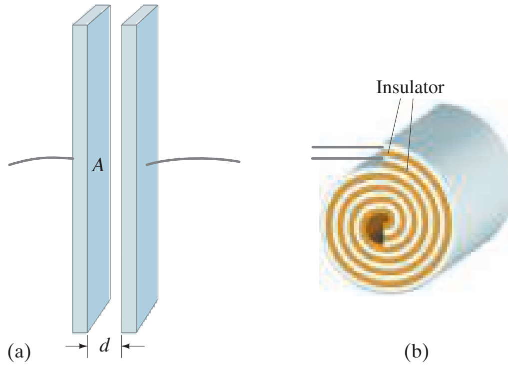


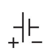
FIGURE 1 Capacitors: diagrams of (a) parallel plate, (b) cylindrical (rolled up parallel plate).

Very tiny capacitors serve as memory for the "ones" and "zeros" of the binary code in the random access memory (RAM) of computers. Capacitors serve many other applications, some of which we will discuss.

A simple capacitor consists of a pair of parallel plates of area A separated by a small distance d (Fig. 1a). Often the two plates are rolled into the form of a cylinder with plastic, paper, or other insulator separating the plates, Fig. 1b. In a diagram, the symbol

 or  [capacitor symbol]

represents a capacitor. A battery, which is a source of voltage, is indicated by the symbol:

 [battery symbol]

with unequal arms.

If a voltage is applied across a capacitor by connecting the capacitor to a battery with conducting wires as in Fig. 2, the two plates quickly become charged: one plate acquires a negative charge, the other an equal amount of positive charge. Each battery terminal and the plate of the capacitor connected to it are at the same potential; hence the full battery voltage appears across the capacitor. For a given capacitor, it is found that the amount of charge Q acquired by each plate is proportional to the magnitude of the potential difference V between them:

$$Q = CV. \tag{1}$$

The constant of proportionality, C, in the above relation is called the **capacitance** of the capacitor. The unit of capacitance is coulombs per volt and this unit is called a **farad** (F). Common capacitors have capacitance in the range of 1 pF $\left(\text{picofarad} = 10^{-12}\,\text{F}\right)$ to $10^3\,\mu\text{F}$ $\left(\text{microfarad} = 10^{-6}\,\text{F}\right)$. The relation, Eq. 1, was first suggested by Volta in the late eighteenth century. The capacitance C does not in general depend on Q or V. Its value depends only on the size, shape, and relative position of the two conductors, and also on the material that separates them.

In Eq. 1, and from now on, we use simply V (in italics) to represent a potential difference, rather than V_{ba}, ΔV, or $V_{\text{b}} - V_{\text{a}}$, as previously. (Be sure not to confuse italic V and C which stand for voltage and capacitance, with non-italic V and C which stand for the units volts and coulombs).

EXERCISE A Graphs for charge versus voltage are shown in Fig. 3 for three capacitors, A, B, and C. Which has the greatest capacitance?

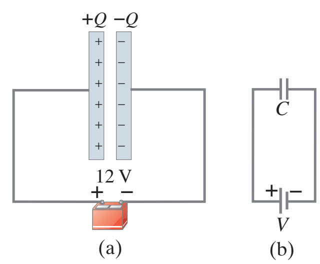
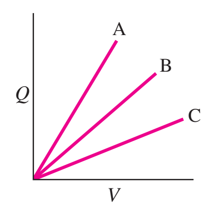


FIGURE 2 (a) Parallel-plate capacitor connected to a battery. (b) Same circuit shown using symbols.

FIGURE 3 Exercise A.



CAUTION
V = potential difference from here on

2 Determination of Capacitance

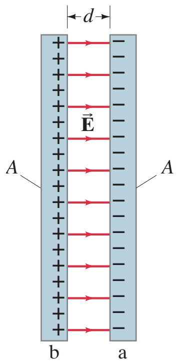


FIGURE 4 Parallel-plate capacitor, each of whose plates has area A. Fringing of the field is ignored.

The capacitance of a given capacitor can be determined experimentally directly from Eq. 1, by measuring the charge Q on either conductor for a given potential difference V.

For capacitors whose geometry is simple, we can determine C analytically, and in this Section we assume the conductors are separated by a vacuum or air. First, we determine C for a parallel-plate capacitor, Fig. 4. Each plate has area A and the two plates are separated by a distance d. We assume d is small compared to the dimensions of each plate so that the electric field $\vec{\mathbf{E}}$ is uniform between them and we can ignore fringing (lines of $\vec{\mathbf{E}}$ not straight) at the edges. The electric field between two closely spaced parallel plates has magnitude $E = \sigma/\epsilon_0$ and its direction is perpendicular to the plates. Since σ is the charge per unit area, $\sigma = Q/A$, then the field between the plates is

$$E = \frac{Q}{\epsilon_0 A}.$$

The relation between electric field and electric potential, as given by Eq. 4a, is

$$V = V_{\mathrm{ba}} = V_{\mathrm{b}} - V_{\mathrm{a}} = -\int_{\mathrm{a}}^{\mathrm{b}} \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}.$$

We can take the line integral along a path antiparallel to the field lines, from plate a to plate b; then $\theta = 180°$ and $\cos 180° = -1$, so

$$V = V_{\mathrm{b}} - V_{\mathrm{a}} = -\int_{\mathrm{a}}^{\mathrm{b}} E\, d\ell \cos 180° = +\int_{\mathrm{a}}^{\mathrm{b}} E\, d\ell = \frac{Q}{\epsilon_0 A}\int_{\mathrm{a}}^{\mathrm{b}} d\ell = \frac{Qd}{\epsilon_0 A}.$$

This relates Q to V, and from it we can get the capacitance C in terms of the geometry of the plates:

$$C = \frac{Q}{V} = \epsilon_0 \frac{A}{d}. \qquad \text{[parallel-plate capacitor]} \qquad \textbf{(2)}$$

Note from Eq. 2 that the value of C does not depend on Q or V, so Q is predicted to be proportional to V as is found experimentally.

EXAMPLE 1 **Capacitor calculations.** (*a*) Calculate the capacitance of a parallel-plate capacitor whose plates are $20\,\mathrm{cm} \times 3.0\,\mathrm{cm}$ and are separated by a 1.0-mm air gap. (*b*) What is the charge on each plate if a 12-V battery is connected across the two plates? (*c*) What is the electric field between the plates? (*d*) Estimate the area of the plates needed to achieve a capacitance of 1 F, given the same air gap d.

APPROACH The capacitance is found by using Eq. 2, $C = \epsilon_0 A/d$. The charge on each plate is obtained from the definition of capacitance, Eq. 1, $Q = CV$. The electric field is uniform, so for the magnitude we can use $E = V/d$. In (*d*) we use Eq. 2 again.

SOLUTION (*a*) The area $A = (20 \times 10^{-2}\,\mathrm{m})(3.0 \times 10^{-2}\,\mathrm{m}) = 6.0 \times 10^{-3}\,\mathrm{m}^2$. The capacitance C is then

$$C = \epsilon_0 \frac{A}{d} = \left(8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\frac{6.0 \times 10^{-3}\,\mathrm{m}^2}{1.0 \times 10^{-3}\,\mathrm{m}} = 53\,\mathrm{pF}.$$

(*b*) The charge on each plate is

$$Q = CV = \left(53 \times 10^{-12}\,\mathrm{F}\right)(12\,\mathrm{V}) = 6.4 \times 10^{-10}\,\mathrm{C}.$$

(*c*) For a uniform electric field, the magnitude of E is

$$E = \frac{V}{d} = \frac{12\,\mathrm{V}}{1.0 \times 10^{-3}\,\mathrm{m}} = 1.2 \times 10^4\,\mathrm{V/m}.$$

(*d*) We solve for A in Eq. 2 and substitute $C = 1.0\,\mathrm{F}$ and $d = 1.0\,\mathrm{mm}$ to find that we need plates with an area

$$A = \frac{Cd}{\epsilon_0} \approx \frac{(1\,\mathrm{F})\left(1.0 \times 10^{-3}\,\mathrm{m}\right)}{\left(9 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)} \approx 10^8\,\mathrm{m}^2.$$

NOTE This is the area of a square 10^4 m or 10 km on a side. That is the size of a city like San Francisco or Boston! Large-capacitance capacitors will not be simple parallel plates.

EXERCISE B Two circular plates of radius 5.0 cm are separated by a 0.10-mm air gap. What is the magnitude of the charge on each plate when connected to a 12-V battery?

PHYSICS APPLIED
Very high capacitance

Not long ago, a capacitance greater than a few mF was unusual. Today capacitors are available that are 1 or 2 F, yet they are just a few cm on a side. Such capacitors are used as power backups, for example, in computer memory and electronics where the time and date can be maintained through tiny charge flow. [Capacitors are superior to rechargable batteries for this purpose because they can be recharged more than 10^5 times with no degradation.] Such high-capacitance capacitors can be made of "activated" carbon which has very high porosity, so that the surface area is very large; one tenth of a gram of activated carbon can have a surface area of $100\,\mathrm{m}^2$. Furthermore, the equal and opposite charges exist in an electric "double layer" about $10^{-9}\,\mathrm{m}$ thick. Thus, the capacitance of 0.1 g of activated carbon, whose internal area can be $10^2\,\mathrm{m}^2$, is equivalent to a parallel-plate capacitor with $C \approx \epsilon_0 A/d = \left(8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(10^2\,\mathrm{m^2}\right)/\left(10^{-9}\,\mathrm{m}\right) \approx 1\,\mathrm{F}$.

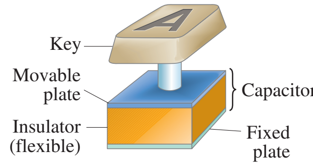


FIGURE 5 Key on a computer keyboard. Pressing the key reduces the capacitor spacing thus increasing the capacitance which can be detected electronically.

PHYSICS APPLIED
Computer key

One type of computer keyboard operates by capacitance. As shown in Fig. 5, each key is connected to the upper plate of a capacitor. The upper plate moves down when the key is pressed, reducing the spacing between the capacitor plates, and increasing the capacitance (Eq. 2: smaller d, larger C). The *change* in capacitance results in an electric signal that is detected by an electronic circuit.

The proportionality, $C \propto A/d$ in Eq. 2, is valid also for a parallel-plate capacitor that is rolled up into a spiral cylinder, as in Fig. 1b. However, the constant factor, ϵ_0, must be replaced if an insulator such as paper separates the plates, as is usual, and this is discussed in Section 5. For a true cylindrical capacitor—consisting of two long coaxial cylinders—the result is somewhat different as the next Example shows.

EXAMPLE 2 **Cylindrical capacitor.** A cylindrical capacitor consists of a cylinder (or wire) of radius R_b surrounded by a coaxial cylindrical shell of inner radius R_a, Fig. 6a. Both cylinders have length ℓ which we assume is much greater than the separation of the cylinders, $R_\mathrm{a} - R_\mathrm{b}$, so we can neglect end effects. The capacitor is charged (by connecting it to a battery) so that one cylinder has a charge $+Q$ (say, the inner one) and the other one a charge $-Q$. Determine a formula for the capacitance.

APPROACH To obtain $C = Q/V$, we need to determine the potential difference V between the cylinders in terms of Q. We can use our earlier result that the electric field outside a long wire is directed radially outward and has magnitude $E = (1/2\pi\epsilon_0)(\lambda/R)$, where R is the distance from the axis and λ is the charge per unit length, Q/ℓ. Then $E = (1/2\pi\epsilon_0)(Q/\ell R)$ for points between the cylinders.

SOLUTION To obtain the potential difference V in terms of Q, we use this result for E in $V = V_\mathrm{b} - V_\mathrm{a} = -\int_\mathrm{a}^\mathrm{b} \vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}}$, and write the line integral from the outer cylinder to the inner one (so $V > 0$) along a radial line:[†]

$$V = V_\mathrm{b} - V_\mathrm{a} = -\int_\mathrm{a}^\mathrm{b} \vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}} = -\frac{Q}{2\pi\epsilon_0 \ell}\int_{R_\mathrm{a}}^{R_\mathrm{b}} \frac{dR}{R}$$

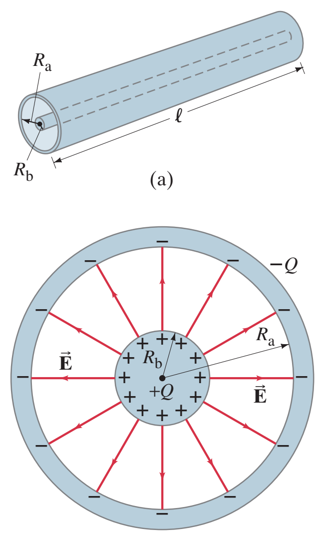
$$= -\frac{Q}{2\pi\epsilon_0 \ell}\ln\frac{R_\mathrm{b}}{R_\mathrm{a}} = \frac{Q}{2\pi\epsilon_0 \ell}\ln\frac{R_\mathrm{a}}{R_\mathrm{b}}.$$

Q and V are proportional, and the capacitance C is

$$C = \frac{Q}{V} = \frac{2\pi\epsilon_0 \ell}{\ln(R_\mathrm{a}/R_\mathrm{b})}. \qquad \text{[cylindrical capacitor]}$$

NOTE If the space between cylinders, $R_\mathrm{a} - R_\mathrm{b} = \Delta R$ is small, we have $\ln(R_\mathrm{a}/R_\mathrm{b}) = \ln[(R_\mathrm{b} + \Delta R)/R_\mathrm{b}] = \ln[1 + \Delta R/R_\mathrm{b}] \approx \Delta R/R_\mathrm{b}$ so $C \approx 2\pi\epsilon_0 \ell R_\mathrm{b}/\Delta R = \epsilon_0 A/\Delta R$ because the area of cylinder b is $A = 2\pi R_\mathrm{b}\ell$. This is just Eq. 2 $(d = \Delta R)$, a nice check.

FIGURE 6 (a) Cylindrical capacitor consists of two coaxial cylindrical conductors. (b) The electric field lines are shown in cross-sectional view.



EXERCISE C What is the capacitance per unit length of a cylindrical capacitor with radii $R_\mathrm{a} = 2.5\,\mathrm{mm}$ and $R_\mathrm{b} = 0.40\,\mathrm{mm}$? (*a*) 30 pF/m; (*b*) −30 pF/m; (*c*) 56 pF/m; (*d*) −56 pF/m; (*e*) 100 pF/m; (*f*) −100 pF/m.

[†]Note that $\vec{\mathbf{E}}$ points outward in Fig. 6b, but $d\vec{\boldsymbol{\ell}}$ points inward for our chosen direction of integration; the angle between $\vec{\mathbf{E}}$ and $d\vec{\boldsymbol{\ell}}$ is 180° and $\cos 180° = -1$. Also, $d\ell = -dr$ because dr increases outward. These two minus signs cancel.

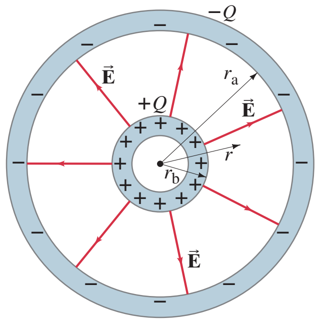


FIGURE 7 Cross section through the center of a spherical capacitor. The thin inner shell has radius r_b and the thin outer shell has radius r_a.

EXAMPLE 3 **Spherical capacitor.** A spherical capacitor consists of two thin concentric spherical conducting shells, of radius r_a and r_b as shown in Fig. 7. The inner shell carries a uniformly distributed charge Q on its surface, and the outer shell an equal but opposite charge $-Q$. Determine the capacitance of the two shells.

APPROACH We can use Gauss's law to show that the electric field outside a uniformly charged conducting sphere is $E = Q/4\pi\epsilon_0 r^2$ as if all the charge were concentrated at the center. However, now we use $V = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$.

SOLUTION We integrate this Equation along a radial line to obtain the potential difference between the two conducting shells:

$$V_{ba} = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{Q}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{1}{r^2}\,dr$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = \frac{Q}{4\pi\epsilon_0}\left(\frac{r_a - r_b}{r_a r_b}\right).$$

Finally,

$$C = \frac{Q}{V_{ba}} = 4\pi\epsilon_0\left(\frac{r_a r_b}{r_a - r_b}\right).$$

PROBLEM SOLVING
Checking with a limiting case

NOTE If the separation $\Delta r = r_a - r_b$ is very small, then $C = 4\pi\epsilon_0 r^2/\Delta r \approx \epsilon_0 A/\Delta r$ (since $A = 4\pi r^2$), which is the parallel-plate formula, Eq. 2.

A single isolated conductor can also be said to have a capacitance, C. In this case, C can still be defined as the ratio of the charge to absolute potential V on the conductor (relative to $V = 0$ at $r = \infty$), so that the relation

$$Q = CV$$

remains valid. For example, the potential of a single conducting sphere of radius r_b can be obtained from our results in Example 3 by letting r_a become infinitely large. As $r_a \to \infty$, then

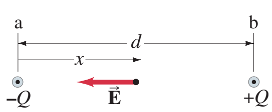
$$V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_b};$$

so its capacitance is

$$C = \frac{Q}{V} = 4\pi\epsilon_0 r_b.$$

In practical cases, a single conductor may be near other conductors or the Earth (which can be thought of as the other "plate" of a capacitor), and these will affect the value of the capacitance.

FIGURE 8 Example 4.



EXAMPLE 4 **Capacitance of two long parallel wires.** Estimate the capacitance per unit length of two very long straight parallel wires, each of radius R, carrying uniform charges $+Q$ and $-Q$, and separated by a distance d which is large compared to R ($d \gg R$), Fig. 8.

APPROACH We calculate the potential difference between the wires by treating the electric field at any point between them as the superposition of the two fields created by each wire. (The electric field inside each wire conductor is zero.)

SOLUTION The electric field outside of a long straight conductor is radial and given by $E = \lambda/(2\pi\epsilon_0 x)$ where λ is the charge per unit length, $\lambda = Q/\ell$. The total electric field at distance x from the left-hand wire in Fig. 8 has magnitude

$$E = \frac{\lambda}{2\pi\epsilon_0 x} + \frac{\lambda}{2\pi\epsilon_0(d - x)},$$

and points to the left (from + to −). Now we find the potential difference between the two wires using $V = -\int_a^b \vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}}$. and integrating along the straight line from the surface of the negative wire to the surface of the positive wire, noting that $\vec{\mathbf{E}}$ and $d\vec{\boldsymbol{\ell}}$ point in opposite directions $(\vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}} < 0)$:

$$V = V_b - V_a = -\int_a^b \vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}} = \left(\frac{\lambda}{2\pi\epsilon_0}\right)\int_R^{d-R}\left[\frac{1}{x} + \frac{1}{(d-x)}\right]dx$$

$$= \left(\frac{\lambda}{2\pi\epsilon_0}\right)\left[\ln(x) - \ln(d-x)\right]\Big|_R^{d-R}$$

$$= \left(\frac{\lambda}{2\pi\epsilon_0}\right)\left[\ln(d-R) - \ln R - \ln R + \ln(d-R)\right]$$

$$= \left(\frac{\lambda}{\pi\epsilon_0}\right)\left[\ln(d-R) - \ln(R)\right] \approx \left(\frac{\lambda}{\pi\epsilon_0}\right)\left[\ln(d) - \ln(R)\right].$$

We are given that $d \gg R$, so

$$V \approx \left(\frac{Q}{\pi\epsilon_0\ell}\right)\left[\ln\left(\frac{d}{R}\right)\right].$$

The capacitance from Eq. 1 is $C = Q/V \approx (\pi\epsilon_0\ell)/\ln(d/R)$, so the capacitance per unit length is given approximately by

$$\frac{C}{\ell} \approx \frac{\pi\epsilon_0}{\ln(d/R)}.$$

3 Capacitors in Series and Parallel

Capacitors are found in many electric circuits. By electric circuit we mean a closed path of conductors, usually wires connecting capacitors and/or other devices, in which charge can flow and which includes a source of voltage such as a battery. The battery voltage is usually given the symbol V, which means that V represents a potential *difference*. Capacitors can be connected together in various ways. Two common ways are in *series*, or in *parallel*, and we now discuss both.

A circuit containing three capacitors connected in **parallel** is shown in Fig. 9. They are in "parallel" because when a battery of voltage V is connected to points a and b, this voltage $V = V_{ab}$ exists across each of the capacitors. That is, since the left-hand plates of all the capacitors are connected by conductors, they all reach the same potential V_a when connected to the battery; and the right-hand plates each reach potential V_b. Each capacitor plate acquires a charge given by $Q_1 = C_1V$, $Q_2 = C_2V$, and $Q_3 = C_3V$. The total charge Q that must leave the battery is then

$$Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V.$$

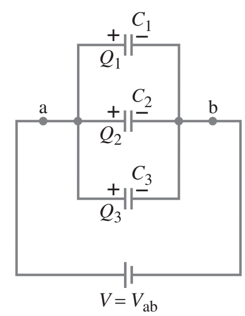


FIGURE 9 Capacitors in parallel: $C_{eq} = C_1 + C_2 + C_3$.

Let us try to find a single equivalent capacitor that will hold the same charge Q at the same voltage $V = V_{ab}$. It will have a capacitance C_{eq} given by

$$Q = C_{eq}V.$$

Combining the two previous equations, we have

$$C_{eq}V = C_1V + C_2V + C_3V = (C_1 + C_2 + C_3)V$$

or

$$C_{eq} = C_1 + C_2 + C_3. \qquad \text{[parallel]} \quad \mathbf{(3)}$$

The net effect of connecting capacitors in parallel is thus to *increase* the capacitance. This makes sense because we are essentially increasing the area of the plates where charge can accumulate (see, for example, Eq. 2).

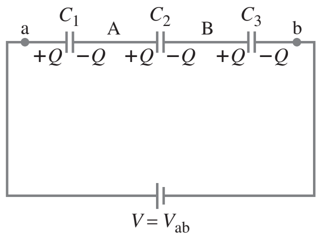


FIGURE 10 Capacitors in series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$.

Capacitors can also be connected in **series**: that is, end to end as shown in Fig. 10. A charge $+Q$ flows from the battery to one plate of C_1, and $-Q$ flows to one plate of C_3. The regions A and B between the capacitors were originally neutral; so the net charge there must still be zero. The $+Q$ on the left plate of C_1 attracts a charge of $-Q$ on the opposite plate. Because region A must have a zero net charge, there is thus $+Q$ on the left plate of C_2. The same considerations apply to the other capacitors, so we see the charge on each capacitor is the same value Q. A single capacitor that could replace these three in series without affecting the circuit (that is, Q and V the same) would have a capacitance C_{eq} where

$$Q = C_{eq}V.$$

Now the total voltage V across the three capacitors in series must equal the sum of the voltages across each capacitor:

$$V = V_1 + V_2 + V_3.$$

We also have for each capacitor $Q = C_1V_1$, $Q = C_2V_2$, and $Q = C_3V_3$, so we substitute for V, V_1, V_2, and V_3 into the last equation and get

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

or

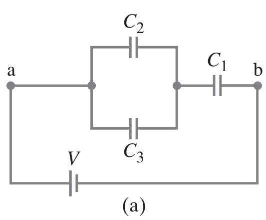
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}. \qquad \text{[series]} \quad \textbf{(4)}$$

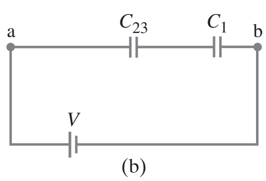
Notice that the equivalent capacitance C_{eq} is smaller than the smallest contributing capacitance.

EXERCISE D Consider two identical capacitors $C_1 = C_2 = 10\,\mu F$. What are the minimum and maximum capacitances that can be obtained by connecting these in series or parallel combinations? (*a*) $0.2\,\mu F, 5\,\mu F$; (*b*) $0.2\,\mu F, 10\,\mu F$; (*c*) $0.2\,\mu F, 20\,\mu F$; (*d*) $5\,\mu F, 10\,\mu F$; (*e*) $5\,\mu F, 20\,\mu F$; (*f*) $10\,\mu F, 20\,\mu F$.

Other connections of capacitors can be analyzed similarly using charge conservation, and often simply in terms of series and parallel connections.

FIGURE 11 Examples 5 and 6.





EXAMPLE 5 **Equivalent capacitance.** Determine the capacitance of a single capacitor that will have the same effect as the combination shown in Fig. 11a. Take $C_1 = C_2 = C_3 = C$.

APPROACH First we find the equivalent capacitance of C_2 and C_3 in parallel, and then consider that capacitance in series with C_1.

SOLUTION Capacitors C_2 and C_3 are connected in parallel, so they are equivalent to a single capacitor having capacitance

$$C_{23} = C_2 + C_3 = 2C.$$

This C_{23} is in series with C_1, Fig. 11b, so the equivalent capacitance of the entire circuit, C_{eq}, is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{23}} = \frac{1}{C} + \frac{1}{2C} = \frac{3}{2C}.$$

Hence the equivalent capacitance of the entire combination is $C_{eq} = \frac{2}{3}C$, and it is smaller than any of the contributing capacitors, $C_1 = C_2 = C_3 = C$.

EXAMPLE 6 **Charge and voltage on capacitors.** Determine the charge on each capacitor in Fig. 11a of Example 5 and the voltage across each, assuming $C = 3.0\,\mu\text{F}$ and the battery voltage is $V = 4.0\,\text{V}$.

APPROACH We have to work "backward" through Example 5. That is, we find the charge Q that leaves the battery, using the equivalent capacitance. Then we find the charge on each separate capacitor and the voltage across each. Each step uses Eq. 1, $Q = CV$.

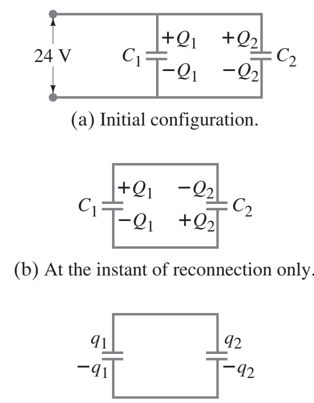
SOLUTION The 4.0-V battery behaves as if it is connected to a capacitance $C_{\text{eq}} = \frac{2}{3}C = \frac{2}{3}(3.0\,\mu\text{F}) = 2.0\,\mu\text{F}$. Therefore the charge Q that leaves the battery, by Eq. 1, is

$$Q = CV = (2.0\,\mu\text{F})(4.0\,\text{V}) = 8.0\,\mu\text{C}.$$

From Fig. 11a, this charge arrives at the negative plate of C_1, so $Q_1 = 8.0\,\mu\text{C}$. The charge Q that leaves the positive plate of the battery is split evenly between C_2 and C_3 (symmetry: $C_2 = C_3$) and is $Q_2 = Q_3 = \frac{1}{2}Q = 4.0\,\mu\text{C}$. Next, the voltages across C_2 and C_3 have to be the same. The voltage across each capacitor is obtained using $V = Q/C$. So

$$\begin{aligned} V_1 &= Q_1/C_1 = (8.0\,\mu\text{C})/(3.0\,\mu\text{F}) = 2.7\,\text{V} \\ V_2 &= Q_2/C_2 = (4.0\,\mu\text{C})/(3.0\,\mu\text{F}) = 1.3\,\text{V} \\ V_3 &= Q_3/C_3 = (4.0\,\mu\text{C})/(3.0\,\mu\text{F}) = 1.3\,\text{V}. \end{aligned}$$

EXAMPLE 7 **Capacitors reconnected.** Two capacitors, $C_1 = 2.2\,\mu\text{F}$ and $C_2 = 1.2\,\mu\text{F}$, are connected in parallel to a 24-V source as shown in Fig. 12a. After they are charged they are disconnected from the source and from each other, and then reconnected directly to each other with plates of opposite sign connected together (see Fig. 12b). Find the charge on each capacitor and the potential across each after equilibrium is established.



(a) Initial configuration.

(b) At the instant of reconnection only.

(c) A short time later.

FIGURE 12 Example 7.

APPROACH We find the charge $Q = CV$ on each capacitor initially. Charge is conserved, although rearranged after the switch. The two new voltages will have to be equal.

SOLUTION First we calculate how much charge has been placed on each capacitor after the power source has charged them fully, using Eq. 1:

$$\begin{aligned} Q_1 &= C_1V = (2.2\,\mu\text{F})(24\,\text{V}) = 52.8\,\mu\text{C}, \\ Q_2 &= C_2V = (1.2\,\mu\text{F})(24\,\text{V}) = 28.8\,\mu\text{C}. \end{aligned}$$

Next the capacitors are connected in parallel, Fig. 12b, and the potential difference across each must quickly equalize. Thus, the charge cannot remain as shown in Fig. 12b, but the charge must rearrange itself so that the upper plates at least have the same sign of charge, with the lower plates having the opposite charge as shown in Fig. 12c. Equation 1 applies for each:

$$q_1 = C_1V' \qquad \text{and} \qquad q_2 = C_2V',$$

where V' is the voltage across each capacitor after the charges have rearranged themselves. We don't know q_1, q_2, or V', so we need a third equation. This is provided by charge conservation. The charges have rearranged themselves between Figs. 12b and c. The total charge on the upper plates in those two Figures must be the same, so we have

$$q_1 + q_2 = Q_1 - Q_2 = 24.0\,\mu\text{C}.$$

Combining the last three equations we find:

$$\begin{aligned} V' &= (q_1 + q_2)/(C_1 + C_2) = 24.0\,\mu\text{C}/3.4\,\mu\text{F} = 7.06\,\text{V} \approx 7.1\,\text{V} \\ q_1 &= C_1V' = (2.2\,\mu\text{F})(7.06\,\text{V}) = 15.5\,\mu\text{C} \approx 16\,\mu\text{C} \\ q_2 &= C_2V' = (1.2\,\mu\text{F})(7.06\,\text{V}) = 8.5\,\mu\text{C} \end{aligned}$$

where we have kept only two significant figures in our final answers.

4 Electric Energy Storage

A charged capacitor stores electrical energy. The energy stored in a capacitor will be equal to the work done to charge it. The net effect of charging a capacitor is to remove charge from one plate and add it to the other plate. This is what a battery does when it is connected to a capacitor. A capacitor does not become charged instantly. It takes time. Initially, when the capacitor is uncharged, it requires no work to move the first bit of charge over. When some charge is on each plate, it requires work to add more charge of the same sign because of the electric repulsion. The more charge already on a plate, the more work required to add additional charge. The work needed to add a small amount of charge dq, when a potential difference V is across the plates, is $dW = V\,dq$. Since $V = q/C$ at any moment (Eq. 1), where C is the capacitance, the work needed to store a total charge Q is

$$W = \int_0^Q V\,dq = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{2}\frac{Q^2}{C}.$$

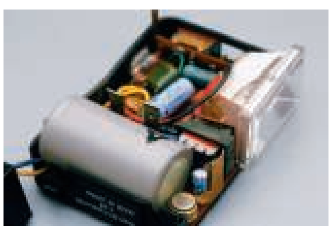
Thus we can say that the energy "stored" in a capacitor is

$$U = \frac{1}{2}\frac{Q^2}{C}$$

when the capacitor C carries charges $+Q$ and $-Q$ on its two conductors. Since $Q = CV$, where V is the potential difference across the capacitor, we can also write

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV. \qquad \textbf{(5)}$$

PHYSICS APPLIED
Camera flash



Tom Pantages

FIGURE 13 A camera flash unit.

EXAMPLE 8 **Energy stored in a capacitor.** A camera flash unit (Fig. 13) stores energy in a 150-μF capacitor at 200 V. (*a*) How much electric energy can be stored? (*b*) What is the power output if nearly all this energy is released in 1.0 ms?

APPROACH We use Eq. 5 in the form $U = \frac{1}{2}CV^2$ because we are given C and V.

SOLUTION The energy stored is

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(150 \times 10^{-6}\,\mathrm{F}\right)(200\,\mathrm{V})^2 = 3.0\,\mathrm{J}.$$

If this energy is released in $\frac{1}{1000}$ of a second, the power output is $P = U/t = (3.0\,\mathrm{J})/\left(1.0 \times 10^{-3}\,\mathrm{s}\right) = 3000\,\mathrm{W}$.

CONCEPTUAL EXAMPLE 9 **Capacitor plate separation increased.** A parallel-plate capacitor carries charge Q and is then disconnected from a battery. The two plates are initially separated by a distance d. Suppose the plates are pulled apart until the separation is $2d$. How has the energy stored in this capacitor changed?

RESPONSE If we increase the plate separation d, we decrease the capacitance according to Eq. 2, $C = \epsilon_0 A/d$, by a factor of 2. The charge Q hasn't changed. So according to Eq. 5, where we choose the form $U = \frac{1}{2}Q^2/C$ because we know Q is the same and C has been halved, the reduced C means the potential energy stored increases by a factor of 2.

NOTE We can see why the energy stored increases from a physical point of view: the two plates are charged equal and opposite, so they attract each other. If we pull them apart, we must do work, so we raise their potential energy.

EXAMPLE 10 **Moving parallel capacitor plates.** The plates of a parallel-plate capacitor have area A, separation x, and are connected to a battery with voltage V. While connected to the battery, the plates are pulled apart until they are separated by $3x$. (a) What are the initial and final energies stored in the capacitor? (b) How much work is required to pull the plates apart (assume constant speed)? (c) How much energy is exchanged with the battery?

APPROACH The stored energy is given by Eq. 5: $U = \frac{1}{2}CV^2$, where $C = \epsilon_0 A/x$. Unlike Example 9, here the capacitor remains connected to the battery. Hence charge and energy can flow to or from the battery, and we can not set the work $W = \Delta U$. Instead, the work can be calculated from the Equation $W = \int \vec{\mathbf{F}} \cdot d\vec{\boldsymbol{\ell}}$.

SOLUTION (a) When the separation is x, the capacitance is $C_1 = \epsilon_0 A/x$ and the energy stored is

$$U_1 = \frac{1}{2}C_1 V^2 = \frac{1}{2}\frac{\epsilon_0 A}{x}V^2.$$

When the separation is $3x$, $C_2 = \epsilon_0 A/3x$ and

$$U_2 = \frac{1}{2}\frac{\epsilon_0 A}{3x}V^2.$$

Then

$$\Delta U_{\text{cap}} = U_2 - U_1 = -\frac{\epsilon_0 AV^2}{3x}.$$

The potential energy decreases as the oppositely charged plates are pulled apart, which makes sense. The plates remain connected to the battery, so V does not change and C decreases; hence some charge leaves each plate $(Q = CV)$, causing U to decrease.

(b) The work done in pulling the plates apart is $W = \int_x^{3x} F\,d\ell = \int_x^{3x} QE\,d\ell$, where Q is the charge on one plate at a given moment when the plates are a distance ℓ apart, and E is the field due to the other plate at that instant. You might think we could use $E = V/\ell$ where ℓ is the separation of the plates. But we want the force on one plate (of charge Q) due to the electric field of the other plate only—which is half by *symmetry*: so we take $E = V/2\ell$. The charge at any separation ℓ is given by $Q = CV$, where $C = \epsilon_0 A/\ell$. Substituting, the work is

$$W = \int_{\ell=x}^{\ell=3x} QE d\ell = \frac{\epsilon_0 AV^2}{2}\int_x^{3x}\frac{d\ell}{\ell^2} = -\frac{\epsilon_0 AV^2}{2\ell}\bigg|_{\ell=x}^{\ell=3x} = \frac{\epsilon_0 AV^2}{2}\left(\frac{-1}{3x} + \frac{1}{x}\right) = \frac{\epsilon_0 AV^2}{3x}.$$

As you might expect, the work required to pull these oppositely charged plates apart is positive.

(c) Even though the work done is positive, the potential energy decreased, which tells us that energy must have gone into the battery (as if charging it). Conservation of energy tells us that the work W done on the system must equal the change in potential energy of the capacitor plus that of the battery (kinetic energy can be assumed to be essentially zero):

$$W = \Delta U_{\text{cap}} + \Delta U_{\text{batt}}.$$

Thus the battery experiences a change in energy of

$$\Delta U_{\text{batt}} = W - \Delta U_{\text{cap}} = \frac{\epsilon_0 AV^2}{3x} + \frac{\epsilon_0 AV^2}{3x} = \frac{2\epsilon_0 AV^2}{3x}.$$

Thus charge flows back into the battery, raising its stored energy. In fact, the battery energy increase is double the work we do.

It is useful to think of the energy stored in a capacitor as being stored in the electric field between the plates. As an example let us calculate the energy stored in a parallel-plate capacitor in terms of the electric field.

The electric field $\vec{\mathbf{E}}$ between two close parallel plates is (approximately) uniform and its magnitude is related to the potential difference by $V = Ed$ where d is the plate separation. Also, Eq. 2 tells us $C = \epsilon_0 A/d$ for a parallel-plate capacitor. Thus

$$\begin{aligned} U &= \tfrac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(E^2 d^2) \\ &= \tfrac{1}{2}\epsilon_0 E^2 Ad. \end{aligned}$$

The quantity Ad is the volume between the plates in which the electric field E exists. If we divide both sides by the volume, we obtain an expression for the energy per unit volume or **energy density**, u:

$$u = \text{energy density} = \tfrac{1}{2}\epsilon_0 E^2. \qquad \textbf{(6)}$$

The *electric energy stored per unit volume in any region of space is proportional to the square of the electric field* in that region. We derived Eq. 6 for the special case of a parallel-plate capacitor. But it can be shown to be true for any region of space where there is an electric field. Note that the units check: for $(\epsilon_0 E^2)$ we have $(\mathrm{C^2/N\cdot m^2})(\mathrm{N/C})^2 = \mathrm{N/m^2} = (\mathrm{N\cdot m})/\mathrm{m^3} = \mathrm{J/m^3}$.

PHYSICS APPLIED
Shocks, burns, defibrillators

Health Effects

The energy stored in a large capacitance can do harm, giving you a burn or a shock. One reason you are warned not to touch a circuit, or the inside of electronic devices, is because capacitors may still be carrying charge even if the external power has been turned off.

On the other hand, the basis of a **heart defibrillator** is a capacitor charged to a high voltage. A heart attack can be characterized by fast irregular beating of the heart, known as *ventricular* (or *cardiac*) *fibrillation*. The heart then does not pump blood to the rest of the body properly, and if it lasts for long, death results. A sudden, brief jolt of charge through the heart from a defibrillator can cause complete heart stoppage, sometimes followed by a resumption of normal beating. The defibrillator capacitor is charged to a high voltage, typically a few thousand volts, and is allowed to discharge very rapidly through the heart via a pair of wide contacts known as "paddles" that spread out the current over the chest (Fig. 14).

FIGURE 14 Heart defibrillator.

5 Dielectrics

In most capacitors there is an insulating sheet of material, such as paper or plastic, called a **dielectric** between the plates. This serves several purposes. First of all, dielectrics break down (allowing electric charge to flow) less readily than air, so higher voltages can be applied without charge passing across the gap. Furthermore, a dielectric allows the plates to be placed closer together without touching, thus allowing an increased capacitance because d is smaller in Eq. 2. Finally, it is found experimentally that if the dielectric fills the space between the two conductors, it increases the capacitance by a factor K which is known as the **dielectric constant**. Thus

$$C = KC_0, \qquad \textbf{(7)}$$

where C_0 is the capacitance when the space between the two conductors of the capacitor is a vacuum, and C is the capacitance when the space is filled with a material whose dielectric constant is K.

TABLE 1 Dielectric Constants (at 20°C)

Material	Dielectric constant K	Dielectric strength (V/m)
Vacuum	1.0000	
Air (1 atm)	1.0006	3×10^6
Paraffin	2.2	10×10^6
Polystyrene	2.6	24×10^6
Vinyl (plastic)	2–4	50×10^6
Paper	3.7	15×10^6
Quartz	4.3	8×10^6
Oil	4	12×10^6
Glass, Pyrex	5	14×10^6
Porcelain	6–8	5×10^6
Mica	7	150×10^6
Water (liquid)	80	
Strontium titanate	300	8×10^6

The values of the dielectric constant for various materials are given in Table 1. Also shown in Table 1 is the **dielectric strength**, the maximum electric field before breakdown (charge flow) occurs.

For a parallel-plate capacitor (see Eq. 2),

$$C = K\epsilon_0 \frac{A}{d} \qquad \text{[parallel-plate capacitor]} \qquad \textbf{(8)}$$

when the space between the plates is completely filled with a dielectric whose dielectric constant is K. (The situation when the dielectric only partially fills the space will be discussed shortly in Example 11.) The quantity $K\epsilon_0$ appears so often in formulas that we define a new quantity

$$\epsilon = K\epsilon_0 \tag{9}$$

called the **permittivity** of a material. Then the capacitance of a parallel-plate capacitor becomes

$$C = \epsilon \frac{A}{d}.$$

Note that ϵ_0 represents the permittivity of free space (a vacuum).

The energy density stored in an electric field E (Section 4) in a dielectric is given by (see Eq. 6)

$$u = \tfrac{1}{2}K\epsilon_0 E^2 = \tfrac{1}{2}\epsilon E^2. \qquad [E \text{ in a dielectric}]$$

EXERCISE E Return to the Chapter-Opening Question and answer it again now. Try to explain why you may have answered differently the first time.

Two simple experiments illustrate the effect of a dielectric. In the first, Fig. 15a, a battery of voltage V_0 is kept connected to a capacitor as a dielectric is inserted between the plates. If the charge on the plates without dielectric is Q_0, then when the dielectric is inserted, it is found experimentally (first by Faraday) that the charge Q on the plates is increased by a factor K,

$$Q = KQ_0. \qquad \text{[voltage constant]}$$

The capacitance has increased to $C = Q/V_0 = KQ_0/V_0 = KC_0$, which is Eq. 7. In a second experiment, Fig. 15b, a battery V_0 is connected to a capacitor C_0 which then holds a charge $Q_0 = C_0V_0$. The battery is then disconnected, leaving the capacitor isolated with charge Q_0 and still at voltage V_0. Next a dielectric is inserted between the plates of the capacitor. The charge remains Q_0 (there is nowhere for the charge to go) but the voltage is found experimentally to drop by a factor K:

$$V = \frac{V_0}{K}. \qquad \text{[charge constant]}$$

Note that the capacitance changes to $C = Q_0/V = Q_0/(V_0/K) = KQ_0/V_0 = KC_0$, so this experiment too confirms Eq. 7.

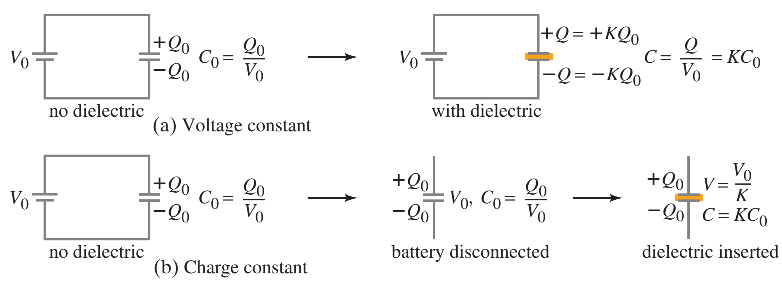


FIGURE 15 Two experiments with a capacitor. Dielectric inserted with (a) voltage held constant, (b) charge held constant.

The electric field when a dielectric is inserted is also altered. When no dielectric is present, the electric field between the plates of a parallel-plate capacitor is given by:

$$E_0 = \frac{V_0}{d},$$

where V_0 is the potential difference between the plates and d is their separation. If the capacitor is isolated so that the charge remains fixed on the plates when a dielectric is inserted, filling the space between the plates, the potential difference drops to $V = V_0/K$. So the electric field in the dielectric is

$$E = E_D = \frac{V}{d} = \frac{V_0}{Kd}$$

or

$$E_D = \frac{E_0}{K}. \qquad \text{[in a dielectric]} \tag{10}$$

The electric field in a dielectric is reduced by a factor equal to the dielectric constant. The field in a dielectric (or insulator) is not reduced all the way to zero as in a conductor. Equation 10 is valid even if the dielectric's width is smaller than the gap between the capacitor plates.

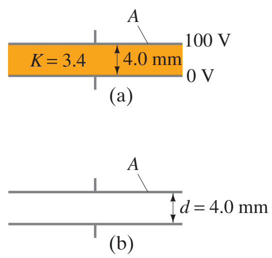


FIGURE 16 Example 11.

EXAMPLE 11 **Dielectric removal.** A parallel-plate capacitor, filled with a dielectric with $K = 3.4$, is connected to a 100-V battery (Fig. 16a). After the capacitor is fully charged, the battery is disconnected. The plates have area $A = 4.0\,\mathrm{m}^2$, and are separated by $d = 4.0\,\mathrm{mm}$. (*a*) Find the capacitance, the charge on the capacitor, the electric field strength, and the energy stored in the capacitor. (*b*) The dielectric is carefully removed, without changing the plate separation nor does any charge leave the capacitor (Fig. 16b). Find the new values of capacitance, electric field strength, voltage between the plates, and the energy stored in the capacitor.

APPROACH We use the formulas for parallel-plate capacitance and electric field with and without a dielectric.

SOLUTION (*a*) First we find the capacitance, with dielectric:

$$C = \frac{K\epsilon_0 A}{d} = \frac{3.4\left(8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(4.0\,\mathrm{m^2}\right)}{4.0 \times 10^{-3}\,\mathrm{m}} = 3.0 \times 10^{-8}\,\mathrm{F}.$$

The charge Q on the plates is

$$Q = CV = \left(3.0 \times 10^{-8}\,\mathrm{F}\right)(100\,\mathrm{V}) = 3.0 \times 10^{-6}\,\mathrm{C}.$$

The electric field between the plates is

$$E = \frac{V}{d} = \frac{100\,\mathrm{V}}{4.0 \times 10^{-3}\,\mathrm{m}} = 25\,\mathrm{kV/m}.$$

Finally, the total energy stored in the capacitor is

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(3.0 \times 10^{-8}\,\mathrm{F}\right)(100\,\mathrm{V})^2 = 1.5 \times 10^{-4}\,\mathrm{J}.$$

(*b*) The capacitance without dielectric decreases by a factor $K = 3.4$:

$$C_0 = \frac{C}{K} = \frac{\left(3.0 \times 10^{-8}\,\mathrm{F}\right)}{3.4} = 8.8 \times 10^{-9}\,\mathrm{F}.$$

Because the battery has been disconnected, the charge Q can not change; when the dielectric is removed, $V = Q/C$ increases by a factor $K = 3.4$ to 340 V. The electric field is

$$E = \frac{V}{d} = \frac{340\,\mathrm{V}}{4.0 \times 10^{-3}\,\mathrm{m}} = 85\,\mathrm{kV/m}.$$

The energy stored is

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(8.8 \times 10^{-9}\,\mathrm{F}\right)(340\,\mathrm{V})^2 = 5.1 \times 10^{-4}\,\mathrm{J}.$$

NOTE Where did all this extra energy come from? The energy increased because work had to be done to remove the dielectric. The work required was $W = \left(5.1 \times 10^{-4}\,\mathrm{J}\right) - \left(1.5 \times 10^{-4}\,\mathrm{J}\right) = 3.6 \times 10^{-4}\,\mathrm{J}$. (We will see in the next Section that work is required because of the force of attraction between induced charge on the dielectric and the charges on the plates, Fig. 17c.)

*6 Molecular Description of Dielectrics

Let us examine, from the molecular point of view, why the capacitance of a capacitor should be larger when a dielectric is between the plates. A capacitor whose plates are separated by an air gap has a charge $+Q$ on one plate and $-Q$ on the other (Fig. 17a). Assume it is isolated (not connected to a battery) so charge cannot flow to or from the plates. The potential difference between the plates, V_0, is given by Eq. 1:

$$Q = C_0 V_0,$$

where the subscripts refer to air between the plates. Now we insert a dielectric between the plates (Fig. 17b). Because of the electric field between the capacitor plates, the dielectric molecules will tend to become oriented as shown in Fig. 17b. If the dielectric molecules are *polar*, the positive end is attracted to the negative plate and vice versa. Even if the dielectric molecules are not polar, electrons within them will tend to move slightly toward the positive capacitor plate, so the effect is the same. The net effect of the aligned dipoles is a net negative charge on the outer edge of the dielectric facing the positive plate, and a net positive charge on the opposite side, as shown in Fig. 17c.

Some of the electric field lines, then, do not pass through the dielectric but instead end on charges induced on the surface of the dielectric as shown in Fig. 17c. Hence the electric field within the dielectric is less than in air. That is, the electric field between the capacitor plates, assumed filled by the dielectric, has been reduced by some factor K. The voltage across the capacitor is reduced by the same factor K because $V = Ed$ (Eq. 4b) and hence, by Eq. 1, $Q = CV$, the capacitance C must increase by that same factor K to keep Q constant.

As shown in Fig. 17d, the electric field within the dielectric E_D can be considered as the vector sum of the electric field $\vec{\mathbf{E}}_0$ due to the "free" charges on the conducting plates, and the field $\vec{\mathbf{E}}_{\text{ind}}$ due to the induced charge on the surfaces of the dielectric. Since these two fields are in opposite directions, the net field within the dielectric, $E_0 - E_{\text{ind}}$, is less than E_0. The precise relationship is given by Eq. 10, even if the dielectric does not fill the gap between the plates:

$$E_D = E_0 - E_{\text{ind}} = \frac{E_0}{K},$$

so

$$E_{\text{ind}} = E_0\left(1 - \frac{1}{K}\right).$$

The electric field between two parallel plates is related to the surface charge density, σ, by $E = \sigma/\epsilon_0$. Thus

$$E_0 = \sigma/\epsilon_0$$

where $\sigma = Q/A$ is the surface charge density on the conductor; Q is the net charge on the conductor and is often called the **free charge** (since charge is free to move in a conductor). Similarly, we define an equivalent induced surface charge density σ_{ind} on the dielectric; then

$$E_{\text{ind}} = \sigma_{\text{ind}}/\epsilon_0$$

where E_{ind} is the electric field due to the induced charge $Q_{\text{ind}} = \sigma_{\text{ind}} A$ on the surface of the dielectric, Fig. 17d. Q_{ind} is often called the **bound charge**, since it is on an insulator and is not free to move. Since $E_{\text{ind}} = E_0(1 - 1/K)$ as shown above, we now have

$$\sigma_{\text{ind}} = \sigma\left(1 - \frac{1}{K}\right) \qquad \textbf{(11a)}$$

and

$$Q_{\text{ind}} = Q\left(1 - \frac{1}{K}\right). \qquad \textbf{(11b)}$$

Since K is always greater than 1, we see that the charge induced on the dielectric is always less than the free charge on each of the capacitor plates.

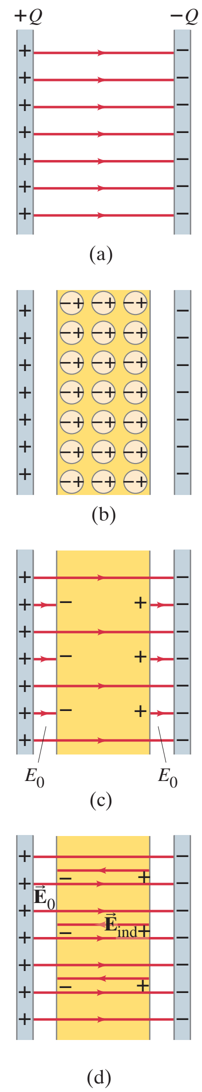


FIGURE 17 Molecular view of the effects of a dielectric.

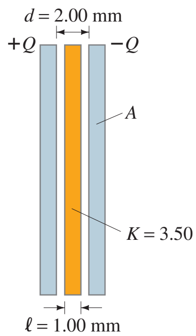


FIGURE 18 Example 12.

EXAMPLE 12 **Dielectric partially fills capacitor.** A parallel-plate capacitor has plates of area $A = 250\,\text{cm}^2$ and separation $d = 2.00\,\text{mm}$. The capacitor is charged to a potential difference $V_0 = 150\,\text{V}$. Then the battery is disconnected (the charge Q on the plates then won't change), and a dielectric sheet $(K = 3.50)$ of the same area A but thickness $\ell = 1.00\,\text{mm}$ is placed between the plates as shown in Fig. 18. Determine (a) the initial capacitance of the air-filled capacitor, (b) the charge on each plate before the dielectric is inserted, (c) the charge induced on each face of the dielectric after it is inserted, (d) the electric field in the space between each plate and the dielectric, (e) the electric field in the dielectric, (f) the potential difference between the plates after the dielectric is added, and (g) the capacitance after the dielectric is in place.

APPROACH We use the expressions for capacitance and charge developed in this Section plus (part e), $V = -\int \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$.

SOLUTION (a) Before the dielectric is in place, the capacitance is

$$C_0 = \epsilon_0 \frac{A}{d} = \left(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(\frac{2.50 \times 10^{-2}\,\text{m}^2}{2.00 \times 10^{-3}\,\text{m}}\right) = 111\,\text{pF}.$$

(b) The charge on each plate is

$$Q = C_0 V_0 = \left(1.11 \times 10^{-10}\,\text{F}\right)(150\,\text{V}) = 1.66 \times 10^{-8}\,\text{C}.$$

(c) Equations 10 and 11 are valid even when the dielectric does not fill the gap, so (Eq. 11b)

$$Q_{\text{ind}} = Q\left(1 - \frac{1}{K}\right) = \left(1.66 \times 10^{-8}\,\text{C}\right)\left(1 - \frac{1}{3.50}\right) = 1.19 \times 10^{-8}\,\text{C}.$$

(d) The electric field in the gaps between the plates and the dielectric (see Fig. 17c) is the same as in the absence of the dielectric since the charge on the plates has not been altered. The result of Example 13 in the "Electric Charge and Electric Field" Chapter can be used here, which gives $E_0 = \sigma/\epsilon_0$. [Or we can note that, in the absence of the dielectric, $E_0 = V_0/d = Q/C_0 d$ $\left(\text{since } V_0 = Q/C_0\right) = Q/\epsilon_0 A$ $\left(\text{since } C_0 = \epsilon_0 A/d\right)$ which is the same result.] Thus

$$E_0 = \frac{Q}{\epsilon_0 A} = \frac{1.66 \times 10^{-8}\,\text{C}}{\left(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.50 \times 10^{-2}\,\text{m}^2\right)} = 7.50 \times 10^4\,\text{V/m}.$$

(e) In the dielectric the electric field is (Eq. 10)

$$E_{\text{D}} = \frac{E_0}{K} = \frac{7.50 \times 10^4\,\text{V/m}}{3.50} = 2.14 \times 10^4\,\text{V/m}.$$

(f) To obtain the potential difference in the presence of the dielectric we use $V = -\int \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$, and integrate from the surface of one plate to the other along a straight line parallel to the field lines:

$$V = -\int \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = E_0(d - \ell) + E_{\text{D}}\ell,$$

which can be simplified to

$$\begin{aligned} V &= E_0\left(d - \ell + \frac{\ell}{K}\right) \\ &= \left(7.50 \times 10^4\,\text{V/m}\right)\left(1.00 \times 10^{-3}\,\text{m} + \frac{1.00 \times 10^{-3}\,\text{m}}{3.50}\right) \\ &= 96.4\,\text{V}. \end{aligned}$$

(g) In the presence of the dielectric, the capacitance is

$$C = \frac{Q}{V} = \frac{1.66 \times 10^{-8}\,\text{C}}{96.4\,\text{V}} = 172\,\text{pF}.$$

NOTE If the dielectric filled the space between the plates, the answers to (f) and (g) would be 42.9 V and 387 pF, respectively.

Summary

A **capacitor** is a device used to store charge (and electric energy), and consists of two nontouching conductors. The two conductors generally hold equal and opposite charges of magnitude Q. The ratio of this charge Q to the potential difference V between the conductors is called the **capacitance**, C:

$$C = \frac{Q}{V} \quad \text{or} \quad Q = CV. \tag{1}$$

The capacitance of a parallel-plate capacitor is proportional to the area A of each plate and inversely proportional to their separation d:

$$C = \epsilon_0 \frac{A}{d}. \tag{2}$$

When capacitors are connected in **parallel**, the equivalent capacitance is the sum of the individual capacitances:

$$C_{eq} = C_1 + C_2 + \cdots. \tag{3}$$

When capacitors are connected in **series**, the reciprocal of the equivalent capacitance equals the sum of the reciprocals of the individual capacitances:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots. \tag{4}$$

A charged capacitor stores an amount of electric energy given by

$$U = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\frac{Q^2}{C}. \tag{5}$$

This energy can be thought of as stored in the electric field between the plates. In any electric field $\vec{\mathbf{E}}$ in free space the **energy density** u (energy per unit volume) is

$$u = \tfrac{1}{2}\epsilon_0 E^2. \tag{6}$$

The space between the conductors contains a nonconducting material such as air, paper, or plastic. These materials are referred to as **dielectrics**, and the capacitance is proportional to a property of dielectrics called the **dielectric constant**, K (nearly equal to 1 for air). For a parallel-plate capacitor

$$C = K\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d} \tag{8}$$

where $\epsilon = K\epsilon_0$ is called the **permittivity** of the dielectric material. When a dielectric is present, the energy density is

$$u = \tfrac{1}{2}K\epsilon_0 E^2 = \tfrac{1}{2}\epsilon E^2.$$

Answers to Exercises

A: A.

B: 8.3×10^{-9} C.

C: (a).

D: (e).

E: (b).